MODELLING ELECTRICITY PRICES: FROM THE STATE OF THE ART TO A DRAFT OF A NEW PROPOSAL

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1. Introduction. Some stylized facts on the electric market and the electricity prices

In the last decades, in order to improve efficiency and reduce electricity prices, following numerous European directives, a progressive liberalization of the electric market has started. This process, which is quite slow due to economies of scale, entry barriers and very high fixed costs faced by those who intend to operate in the energy markets, is in continuous development in some countries, whereas it is already completed in others. Electricity prices will then be determined on the basis of contracts on regular markets, where there is no possibility for arbitrage. In these markets supply will increase or decrease to meet the demand, whose curve results in being inelastic, therefore not much sensitive to price variations.
The large body of literature on electricity prices includes studies with different aims and methodologies depending on the temporal horizon being studied. In the long run the study of the behaviour of electricity spot prices is important for profitability analysis and for power planning, whereas in the medium run it is typically used to obtain a forecast distribution in order to price derivative contracts. The evaluation of derivatives is made on the basis of spot prices, meaning that the price is determined by the market. In this survey we concentrate on those studies whose focus is the determination of trends in electricity spot prices and the forecast of these prices within the short run (day/week-ahead).

Electricity is a particular commodity, characterised by a high variability; this is mainly due to the fact that electric energy cannot be stored, unless through costly and economically unsustainable methods. Only water reserves can be considered as a substitute method to manage the creation of electricity. From the results of numerous studies it does emerge that in Scandinavian countries or in the United States, in which these reserves are abundant, electricity prices show lower peaks due to the possibility of greater flexibility in the creation phase. Therefore, electricity has to be considered as an instantaneous consumption good. A second element capable of influencing prices is the fact that transmission networks are never perfect. Price variation among the different areas occur due to transmission, maintenance and plant costs. Possible overloads and potential faults or technical errors, that could in extreme cases lead to the system blackout, must then be considered in addition to these network problems.

In such a complex framework, the link between price and consumption is extremely difficult to analyse. Consumption, although having a clearly less volatile trend compared to spot prices, presents the same cyclical behaviour. We can therefore state that demand elasticity is very low, but prices are strongly influenced by the level of consumption. High levels of consumption are in fact the determinant of peaks in prices. The increase in demand determines the use of more expensive energetic resources in the production of electricity. In other words, the growth of consumption and therefore of volumes to be produced increases the marginal costs of production, which will rise exponentially depending on the use of nuclear, hydrogen, coal, oil or gas (see Figure 1).
Price cyclicity and electricity demand represent a complex issue.

First of all, electric markets exhibit three different types of seasonality. The first is linked to the greater use of artificial light and heating in the winter, and to the growing use of air conditioning in the summer. The second type of seasonality is weekly and is due to the changes in consumption among weekdays and weekends. Finally, we observe an intra-daily periodicity, which refers to variations between day and night and during the different stages of the day, in which generally we can identify two hot spots. Moreover, it is crucial to take into account that habits and climate conditions change among different countries. Seasonality needs therefore to be continuously focused on each market that has to be analysed. Furthermore, there are other relevant factors of distortion such as extreme temperatures, environmental disasters, particular social events and technical problems previously mentioned, as for example faults in generators.

The combination of the characteristics of the electricity market and the shift from regulated prices to market-determined prices has resulted in a significant increase of electricity price volatility, exemplified by occasional spikes. In fact, electricity spot prices show an extremely high daily average volatility, which varies between 10% and 50%, depending on the markets considered and on price levels, whereas oil and gas volatilities are 3% and 5%, respectively. The search of the best method to model and explain the trend of spot prices, in order to insure producers and consumers from sudden increases, has become in the last years a very relevant issue for the academic world. However, despite the large number of papers published on this topic, there is no clear empirical evidence supporting a specific theoretical model.

The primary goal of this work is to propose a review of the economic literature on empirical electricity spot price analysis. Attention will be drawn on the methodological aspects, mainly economic and statistical, for evaluating the model performance based on estimation/forecast errors of spot prices. Moreover, it is worth noting that the available models are mostly for univariate analysis and that empirical studies mainly concentrate on the Nord Pool, that is the most mature power market in Europe. Because market structures and price dynamics differ
widely across regions, our review will devote special attention to the methodologies applied in different markets. Finally, structural methods of analysis, which are most appropriate for the determination of forward and futures prices, will not be considered here.

The available studies (see Table 1 for a summary) may be classified in terms of the applied methodology. With this respect, three broad classes emerge:

1. **Autoregressive Models**, such as ARMA (AutoRegressive Moving Average), ARX (AutoreRressive with eXogenous inputs), PAR (Periodic AutoRegressive)

2. **Jump and Regime Switching Models**, such as ARJ including Jump having Poisson o Normal distribution, TAR (Threshold AutoRegressive) having a non linear mechanism, which shifts prices from a normal regime (mean reverting) to one with high prices, whose threshold is predetermined, MS (Markov Switching) at two or three regimes, whose threshold is represented by an unobservable random variable.

3. **Volatility Models**, of type ARCH (AutoRegressive Conditional Heteroskedasticity), GARCH (Generalised ARCH), MGARCH (Multivariate GARCH), suitable for describing volatility in a price heteroskedasticity framework (variance changing with time).
Table 1: Recent studies on electricity prices

<table>
<thead>
<tr>
<th>Article</th>
<th>Class</th>
<th>Model Description</th>
<th>Market</th>
<th>Frequency</th>
<th>P. Transformation</th>
<th>Exogenous Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyriel De Jong (2006)</td>
<td>2, 3</td>
<td>Regime Switching, Mean Reverting, Stochastic Jumps, GARCH Models</td>
<td>Nord Pool, EEX, APX, Powernext, EXAA, OMEL, PJM, NEPOOL.</td>
<td>Hourly</td>
<td>Logarithm</td>
<td>None</td>
</tr>
<tr>
<td>Adam Misiorek, Stefan Trueck and Rafal Weron (2006)</td>
<td>1, 2, 3</td>
<td>AR, ARMAX, TAR, TARX, GARCH, Regime Switching Models</td>
<td>CalPX (California Power Exchange)</td>
<td>Hourly</td>
<td>Logarithm</td>
<td>System Load, Temperature, Power Plan Availability</td>
</tr>
<tr>
<td>Rafal Weron and Adam Misiorek (2006)</td>
<td>1, 2</td>
<td>AR, ARX, Threshold ARX Models</td>
<td>Nord Pool</td>
<td>Hourly</td>
<td>Logarithm</td>
<td>Temperature</td>
</tr>
<tr>
<td>Rafał Weron, Inge Simonsen and Piotr Wilman (2003)</td>
<td>2</td>
<td>Mean Reverting and Jump Diffusion Models</td>
<td>Nord Pool</td>
<td>Daily</td>
<td>First Difference</td>
<td>None</td>
</tr>
<tr>
<td>Michael Bierbrauer, Stefan Truck and Rafał Weron (2004)</td>
<td>2</td>
<td>Two and three Regime Switching Models</td>
<td>Nord Pool</td>
<td>Daily</td>
<td>Logarithm</td>
<td>None</td>
</tr>
<tr>
<td>Alvaro Cartea and Marcelo G. Figueroa (2005)</td>
<td>2</td>
<td>Mean Reverting and Jump Diffusion Models</td>
<td>England and Wales Market</td>
<td>Daily</td>
<td>Log Return</td>
<td>None</td>
</tr>
<tr>
<td>Andrew C. Worthington, Adam Kay Spratley and Helen Higgins (2002)</td>
<td>3</td>
<td>M GARCH Model</td>
<td>NEM (Australian National Electricity Market): NSV, QLD, SA, SNO, VIC.</td>
<td>Daily</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Graeme Guthrie and Steen Videbeck (2007)</td>
<td>1</td>
<td>PAR Model, Principal Components</td>
<td>NZEM (New Zealand Electricity Market)</td>
<td>Half Hourly</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Graeme Guthrie and Steen Videbeck (2002)</td>
<td>1, 2</td>
<td>PAR, State Space Models</td>
<td>NZEM (New Zealand Electricity Market)</td>
<td>Half Hourly</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Julia Popova (2004)</td>
<td>1</td>
<td>Spatial Error Models</td>
<td>PJM</td>
<td>Hourly</td>
<td>None</td>
<td>System Load, Temperature, Forward (t-1)</td>
</tr>
<tr>
<td>Michel Culot, Valérie Goffin, Steve Lawford, Sébastien de Menten (2006)</td>
<td>2</td>
<td>Mean Reverting Jump Diffusion, non parametric Models</td>
<td>APX (Amsterdam Power Exchange)</td>
<td>Daily, Hourly</td>
<td>Logarithm</td>
<td>None</td>
</tr>
</tbody>
</table>

Notes. The articles included in the table are classified according to: author, class (1=autoregressive; 2=jump and regime switching; 3=volatility), model applied, reference market, frequency of data, transformation made on prices and exogenous variables used.
2. Modelling electricity spot prices. What does the literature say?

In the following we will summarize the main features of the literature that addresses the issue of evaluating the performance of different models and methodologies applied to the analysis of spot electricity prices and their short run forecasting. Since our goal is to provide a picture of the current state of the art we will illustrate in detail only a selection of recent papers that appear to be very interesting both from the methodological point of view and also in terms of the empirical evidence they provide. On the basis of the evidence coming from the survey we propose in the next section the adoption of a “new” methodological framework for electricity prices analysis and forecasting.

We refer to the Dynamic Factor Models; they have been introduced in the late ‘70s for macroeconomic analysis but they represent a “new” approach to modelling the electricity market in the sense that they never have been used for this purpose.

2.1. General contributions

Point and Interval Forecasting of Spot Electricity Prices: Linear vs. Non-Linear Time Series Models, Adam Misiorek, Stefan Trueck, and Rafał Weron (2006)

An overview of all the candidate models suitable to describe the features of the electricity market is provided by Misiorek, Trueck, and Weron (2006). The aim of their paper is to assess the short-term point and interval forecasting performance of different time series models of the electricity spot market during normal (calm), as well as extremely volatile, periods.

Since the authors want to mimic a typical practitioner praxis, adopting a truly real time forecasting approach, they choose as test ground the California power market, that offers freely accessible high quality electricity price and load data; moreover this is a quite interesting market, since it provides the ideal framework for studying those behaviours typically leading to a market crash (really occurred in winter 2000/2001).

After reviewing the most diffuse time series based modeling approaches for electricity spot prices the authors specify a set of competitor models:

- AR/ARX: linear autoregression models eventually incorporating (X components) exogenous/fundamental factors (the system load in particular),
- AR/ARX-GARCH,
- TAR/TARX (non-linear, threshold regime-switching)
- Markov models with a latent regime-switching variable

Paper retracted upon the request of the author.
The time series of hourly system prices, system-wide loads and day-ahead load forecasts was constructed using data obtained from the UCEI institute and the California independent system operator CAISO, for the calibration period July 5, 1999 – April 2, 2000; the period April 3 – December 3, 2000 was used for out-of-sample testing.

The empirical evidence is again in favour of regime-switching models, but in their simpler form: TAR/TARX models outperform their linear counterparts, both in point and interval forecasting, but simple ARX models reveal a quite encouraging forecasting performance. On the other side, an additional GARCH component generally decreases point forecasting efficiency so that GARCH-inspired specifications do not outperform the relatively simple ARX approach.

The primacy of TARX models emerges both within the point forecasts framework and in the interval forecasting one; in the latter the non-linear Markov regime-switching model systematically underestimated the range of possible next-day electricity prices and yielded the worst results of all tested models.


In the paper by De Jong (2006) the focus is mainly on the existence of typical occasional spikes that are the main source of the large volatility affecting the electricity spot prices and because of their importance are usually incorporated into appropriate pricing, portfolio, and risk management models. Energy markets seem to suffer a level of uncertainty far larger than other commodity markets. Being electricity not storable, spot prices ultimately depend on local and temporal supply and demand conditions. In fact, on one side, large industrial customers usually can not vary their power demand in response to market prices, whereas on the other most power plants can gear up or gear down generation only with a significant time lag.

This low level of flexibility is the main determinant of occasional extreme price spikes, which revert within hours or days to their “standard” level.

In the light of this, the investigation on the nature of power spikes in a number of different markets becomes a relevant line of research.

In particular the author makes a comparison among different time-series models aimed at capturing the dynamics of these disruptive spot prices:

- A standard mean reverting structure which is a simple AR(1) model
- A stochastic Poisson jumps model
- A Markov switching regime model with stochastic Poisson jumps
- A Markov switching model with three regimes: “Normal”-mean reverting, Up and Down
- A Markov switching regime model with independent spikes
- A threshold model
All these models have in common that the spot price (actually a day-ahead price), $P(t)$, is divided into a predictable component $f(t)$ and a stochastic component $X(t)$:

$$p(t) = \ln P(t) = f(t) + X(t).$$

The first component, $f(t)$, accounts for predictable regularities, and typically is a deterministic function of time. The stochastic second component $X(t)$, that is the log spot price from which predictable trends have been removed, is the more interesting and triggers the most of the specification effort by the author.

All the regime switching models above are used to evaluate whether the price spikes should be treated as abnormal and independent deviations from the ‘normal’ price dynamics or whether they form an integral part of the price process.

The empirical application is referred to six day-ahead markets in Europe (Nord Pool Elspot-Scandinavia, EEX-Germany, APX-Netherlands, Powernext-France, EXAA-Austria, OMEL-Spain) and two in the US (PJM-US and New England Pool-US).

As for the empirical evidence the paper concludes that, although they have a limited parameterization, regime-switching models are able to capture the price dynamics significantly better than a GARCH(1,1) model, a jump-model and a threshold model in the eight different markets.

The regime-switching model that strongly looks like a traditional jump model yields the best fit on average, but it is worth noting that there exist significant differences among the markets probably due to the different shares of hydro-power in the total supply stack: in fact hydro-power serves as an indirect means to store electricity, which has a dampening effect on spikes.

### 2.2. Autoregressive models

*High Frequency Electricity Spot Price Dynamics: An Intra-Day Markets Approach, Graeme Guthrie and Steen Videbeck (2002)*

Guthrie and Videbeck develop a new approach to understanding the behavior of high frequency electricity spot prices. Their approach treats electricity delivered at different times of the day as different commodities, while recognizing that these commodities may be traded on a small number of intra-day markets. They first present a detailed analysis of the high frequency dynamics of prices at a key New Zealand node. The analysis, which includes the use of a periodic autoregression model, suggests to consider electricity as multiple commodities, and also reveals intrinsic correlation properties that indicate the existence of distinct intra-day markets. Conventional models cannot adequately capture properties that have important implications for derivative pricing and real options analysis. Guthrie and Videbeck therefore
extend the literature by introducing a state-space model of high frequency spot prices that preserves this intra-day market structure.

The authors used a periodic autoregression model which impacts on both derivative pricing and real options analysis. The periodic autoregression model is used to value electricity derivatives with payoffs depending on high frequency spot price dynamics. The PAR’s principal limitation is the large number of parameters which need to be estimated.

Rather than develop the PAR model, they pursued an alternative approach involving a state-space model. This is easily motivated from the intra-day market structure, and has the additional advantage of requiring a relatively small number of parameters to be estimated. It divides the day into distinct periods based on the correlation structure.

The analysis revealed that daily time series of the prices of these commodities exhibit heterogeneous behavior. Further, the presence of remarkable structure suggests the existence of a small number of intra-day spot markets for electricity. The data suggest also that the structure of intra-day markets varies between weeks and weekends, and across seasons.

Future research will reveal whether these patterns are stable over time and the extent to which they appear in other electricity spot markets. The authors ignored this seasonality in intra-day market structure when estimating the state-space model in order to keep their model to manageable proportions. If more efficient means of estimating the state-space model can be found, then this extra-level of detail can be incorporated into dynamic models of spot prices.

**Spatial Pattern in Modelling Electricity Prices: Evidence From the PJM Market, Julia Popova (2004)**

This paper focuses on the evolution of electricity prices in deregulated market. The author formulates a model that takes into account the spatial features of a network of a market. The model is applied to equilibrium electricity spot prices of the PJM market. This paper addresses the issue of modelling spot prices, because spot prices are one of the key factors in strategic planning and decision support systems of a majority of market players, and are the underlying instrument of a number of electric power derivatives. The goal of the paper is to propose a model for electricity spot price dynamics that takes into account the key characteristics of electricity price formation in the PJM interconnection such as seasonality, weather-dependence, trading in the day-ahead market and spatial attributes of the distribution system.

The novelty of this approach is the utilization of the spatial feature of the PJM market which is divided into twelve transmission zones. The PJM interconnection’s pricing mechanism and price data availability is designed in such a way as to allow considering each zone as a
hypothetical generating unit. Both forward and spot prices are reported for each hypothetical producer hourly. This facilitates a high-frequency empirical analysis taking into account spatial characteristics of the interconnection. Consequently, the author assumes that the electricity spot price can be represented as a function of its lagged values, the forward price, weather conditions, and demand, which is equal to load. Popova assumes also that there is a unique price generating process, but the disturbances are spatially correlated due to the grid topology and the omitted variables problem.

An empirical analysis indicates that the problem of unobserved spatial correlation in the network can be modelled by the Spatial Error providing an additional insight about the spot electricity prices in this market. The spatial aspect plays an essential role in electricity prices formation and ignoring the spatial characteristics and the grid topology may cause biased results and vague conclusions. The problem of unobserved spatial correlation in the grid can be modelled by the SEM. Strong spatial correlation is supported by the estimating results as well as by the testing procedure. Though the estimation of the “spatial” parameter is of little interest, it helps to bring out consistent estimates of explanatory variables. Therefore, the more robust estimates and inference can be drawn.

Despite its attractiveness, the Spatial Error Model is not the only method available to model the electricity prices and derivatives. Future of electricity price modelling may be oriented towards models incorporating finer components and an additional information about the network topology, weather conditions and connections between the PJM zones. The additional information can be utilized either by spatial approach or by other modelling methods.

**Point and interval forecasting of wholesale electricity prices: Evidence from the Nord Pool market, Rafal Weron and Adam Misiorek (2006)**

Rafal Weron and Adam Misiorek assess the short-term forecasting power of different time series models in the Nord Pool electricity spot market. Four five-week periods were selected, which roughly correspond to the months of February, May, August and November. Given this choice, the authors are able to evaluate the performance of the models for all seasons of the year and the large out-of-sample interval allows for a more thorough analysis of the forecasting results when compared to the investigations which are typically used in the literature considering single-week test samples.

The models for electricity spot prices considered by the authors include linear and non-linear autoregressive time series with and without additional fundamental variables. The only exogenous information is the air temperature, since generally this is the most influential weather
variable on electricity prices. The models were tested on a time series of hourly system prices and temperatures from the Nordic power market.

Weron and Misiorek evaluate the accuracy of both point and interval predictions; the latter are specifically important for risk management purposes, where one is more interested in predicting intervals for future price movements than simply point estimates. The authors investigate the quality of the predictions, both in terms of the Mean Weekly Error (for point forecasts) and in terms of the nominal coverage of the models with respect to the true coverage (for interval predictions). They find evidence that non-linear models outperform their linear counterparts and that the interval forecasts of all models are overestimated in the relatively non-volatile periods. During relatively calm periods the AR and spike pre-processed AR (p-AR) models generally yielded better point forecasts than their competitors, with p-AR being slightly better than the pure AR specification. However, during volatile weeks of May 2004 for example, the TAR model was the best. Regarding interval forecasts, they found that the estimated 90% and especially the 99% confidence intervals (CI) of the linear models are clearly too narrow for the volatile period.

Better results are obtained for the TAR model, especially for the 90% CI. However, it predicts slightly too narrow 99% intervals and significantly too wide 50% intervals.

Moreover, the authors found that during relatively calm periods for all models almost all confidence intervals include the actual market clearing price (MCP) value. This is especially true for the 90% and 99% intervals, but even for the 50% CIs deviations from the actual MCP are rarely large enough to exclude the price from the interval. This is in contrast to the results for the California power market, where the TAR model yielded acceptable interval forecasts for the whole test sample. A possible reason for such a behavior could be temporal dependence (or “non-whiteness”) in the model residuals. Whether this is true has yet to be investigated.

**Electricity Spot Price Dynamics: Beyond Financial Models, Graeme Guthrie and Steen Videbeck (2007)**

This study shows that some important properties of electricity spot prices cannot be captured by the statistical models, which are commonly used to model financial asset prices. Using more than eight years of half-hourly spot price data from the New Zealand Electricity Market, Guthrie and Videbeck find that the half-hourly trading periods fall naturally into five groups corresponding to the overnight off-peak, the morning peak, daytime off-peak, evening peak, and evening off-peak. The starting point for the analysis is to acknowledge that its non-storability means that electricity traded at a particular time of the day is a distinct commodity, quite different from electricity traded at different times. The prices in different trading periods within
each group are highly correlated with each other, yet the correlations between prices in different groups are lower. Financial models, which are currently applied to electricity spot prices, are incapable of capturing their behavior. On the contrary, the authors use a periodic autoregression to model prices, showing that shocks in the peak periods are larger and less persistent than those in off-peak periods, and that they often reappear in the following peak period. In contrast, shocks in the off-peak periods are smaller, more persistent, and die out (perhaps temporarily) during the peak periods.

Guthrie and Videbeck illustrate a new approach to modelling electricity prices, the use of periodic autoregressions, because current approaches cannot capture this behavior either. A simple AR process, which ignores the different behavior of prices in different trading periods, can be calibrated to capture the low persistence evident in peak periods, or the greater persistence in off-peak periods, but not both simultaneously. Nor can it capture the reappearance of shocks later in the day when they first appear. The periodic autoregression used in this paper could be used to value electricity derivatives with payoffs depending on high frequency spot price dynamics. The PAR’s main limitation, however, is the large number of parameters to be estimated. For example, with half-hourly trading periods each of the 48 equations has 48 slope coefficients, in addition to the coefficients of various dummy variables. However, much of the dynamic structure would remain if only a subset of the lagged prices (for example 1, 2, 47 and 48 lags) is used.

Parsimonious specifications might allow to introduce jumps and other relevant properties into the price process, although this line of research is not investigated in this work.

2.3. Jump and regime-switching models


After reviewing the stylized facts about power markets, the authors build up a model which takes into account the well-known peculiar statistical properties of electricity spot prices.

The first step of their analysis is to remove the seasonal components of the spot price, which are due to the fluctuations in demand, both at the annual level (due to climate condition) and at the weekly level (troughs over the weekends). The second step is to model the stochastic part of the deseasonalised series with a typical Ornstein-Uhlenbeck process, allowing for mean reversion and a volatility regime driven by a standard Brownian motion.

Naturally, it is the “jumpy” characteristics of prices (after a jump they tend to remain high for several time periods, hours, sometimes even days) which requires a regime-switching model. Working on average daily spot prices from the Nord Pool power exchange since January 1,
1997 until April 25, 2000, the authors propose and fit various models that exhibit mean reversion and assess their performance by comparing simulated and market prices.

The models are:
- A two-regime model with Gaussian spikes
- A two-regime model with lognormal spikes
- A two-regime model with Pareto spikes

The evidence is in line with De Jong (2006) in supporting the changing regime approach: in fact all the models seem to produce estimates for transition probabilities that can be interpreted according to market behaviour. Simulated price trajectories show high correlation with real price data and the parameters estimates are only slightly biased.

However, a critical point does emerge: in some cases the number of price spikes or extreme events generated by simulation of the estimated models is higher than in real price data. This evidence could be explained in terms of a structural drawback of the two-regime models, which are not able to distinguish a new current spike from the reversion following a past spike. Otherwise, this fact could reveal a kind of hypersensitivity typical of such a model category.

While the number of extreme events is overestimated in all models, their estimated magnitude is smaller than the real one in the normal and lognormal models and greater within the Pareto-based one. To sum up, this exercise seems to provide strong support in favour of the three regime model already pointed out also in De Jong (2006). A comparison among different regime switching structures for spot prices modelling with reference to the Nordic power market is conducted also by Michael Bierbrauer, Stefan Truck, and Rafal Weron (2006).

*Modeling electricity prices with regime switching models*, Michael Bierbrauer, Stefan Truck, and Rafal Weron (2004)

After reviewing the stylized facts about power markets, the authors propose various models to spot prices. They analyze and model the logarithm of the deseasonalized average daily spot prices from the Nord Pool power exchange since January 1997 until April 25, 2000 and address the issue of modelling spot electricity prices with regime switching models. The price behavior of spot electricity prices is modelled by dividing the time series into separate phases or regimes with different underlying processes. A jump in electricity prices is considered as a change to another regime. The switching mechanism is assumed to be governed by a random variable that follows a Markov chain with different possible states. Thus, there exists an unobservable variable in the time series that switches between a certain number of states which themselves
are driven by independent stochastic processes. Additionally, there is a probability law that governs the transition from one state to another.

The authors start considering the simplest model with two possible states. The two-regime model distinguishes between a base regime and a spike regime. They assume that base regime is governed by a mean-reverting process and they try different types of distributions for the spike regime. As suggested in the literature, Gaussian and Lognormal distribution are used. Since spikes happen very rarely but they usually are of large magnitude, they consider also heavy-tailed distributions (Pareto) for the spike regime.

Clearly, the variety of regime-switching models is due to the possibility of choosing the number of regimes. Following Huisman and Mahieu (2003), they propose a regime switching model with three possible regimes. The idea behind their specification differs significantly from the previous two-state models. They identify three possible regimes:

- a regime modeling the "normal" electricity price dynamics,
- an initial jump regime for a sudden increase or decrease in price
- a regime that describes how prices move back to the normal regime after the initial jump has occurred.

This definition implies that the initial jump regime is immediately followed by the reverting regime and then moves back to the base regime.

While most spikes only last for one day, there are periods where the prices exhibit three or more extreme events in a row, a behaviour that could be considered as consecutive spikes. In contrast to the two-regime models, the three-regime model does not allow for consecutive spikes (or remaining at a different price level for two or more periods after a jump).

Eventually they assess the performance of the models by comparing simulated and market prices.

The main finding is that the models produce estimates for transition probabilities that can be interpreted according to market behavior. Simulated trajectories show close similarities with real price data. However, the number of price spikes or extreme events produced by simulations of the estimated models is higher than what could be observed in real price data. This is especially true for the two-regime models, where consecutive spikes have a higher probability than in the three-regime model.

Pricing in Electricity Markets: a mean reverting jump diffusion model with seasonality, Alvaro Cartea and Marcelo G. Figueroa (2005)

In this paper Cartea and Figueroa present a mean-reverting jump diffusion model for the electricity spot price and derive the corresponding forward in closed-form. They have analysed
electricity spot prices in the market of England and Wales. The introduction of NETA changed in a fundamental way the behaviour of this market introducing competition and price variations. However, its implementation only took place in March 27, 2001, resulting in not enough data to estimate or test the available models. Driven by this lack of data, the authors proposed a spot-based model from which it is also possible to extract in closed-form the forward curve. Both historical spot data as well as market forwards data are then used to calibrate the parameters of the model. Regarding the calibration of the model, the authors have circumvented a known drawback in electricity spot-based models, that is the overwhelming dependence on a large number of parameters to estimate. As the market evolves and more data becomes available, it will be possible to estimate all the parameters more robustly, as already pointed out by some papers which have analysed more mature markets. The authors are able to reduce the number of parameters to be estimated, using a ‘hybrid’ approach which estimates some parameters from historical spot data and the remaining from market forward prices. It can be argued that this is an arbitrary choice, since calibrating to a market curve starting at a different point might yield different parameters. Even if this were the case, this is not a serious flaw. This would imply recalibrating the forward curve with respect to a different market curve. In a dynamic hedging-strategy, this could be done as many times as necessary, depending on the exposure and the nature of the contract.

As to the output of the model, the simulated price path resembles accurately the evolution of electricity spot prices as observed in this market. With regards to the forward curve shown, it succeeds in capturing changing convexities, which is a serious flaw in models that fail to incorporate seasonality effects or an appropriate number of other determinants.

Finally, the robust evidence of fat tails in the distributions of electricity returns, together with the complexities on the calibration of these spot-based models and the existing problem of the exiguous data in this market, suggests the exploration of different alternatives. An interesting line of work to pursue involves models departing from the Gaussian distribution, as for instance those based on Levy processes.


Since regime switching structures (allowing for mean reversion and long memory) seem to provide a qualified framework in order to correctly model the behaviour of spot prices, some papers tried to provide the different regimes with an economic justification as well.

For this purpose Haldrup and Nielsen (2006) focus on the multilateral electricity price behaviour across regions with physical exchange of power and check the hypothesis that
different regimes reflect congestion and non-congestion periods and that the direction of possible congestion episodes produces significant effects on the price dynamics.

In the authors’ view, a situation where no grid congestions (or grid bottlenecks) exist across neighboring interconnectors will be characterised by a single identical price across the areas with no congestions. However, when the transmission capacity in a sector of the grid is not sufficient, a congestion will arise and the market system will establish different price areas, with the higher price (positive price difference with respect to the other area prices) expressed by the region with excess demand of power.

Price differentials among different areas reflect disequilibria between demand and supply in sub-sectors of the grid: the bidding area with the largest price is the area with excess demand.

As a consequence, an electricity market (for example Nord Pool) may be partitioned into separate bidding areas which become also separate price areas when the contractual flows between them exceed the capacity allocated by the transmission system operators for spot contracts. Three different states can be arise: non-congestion and congestion periods with excess demand in the one or the other region.

To explore this issue the authors focus on separate prices bilaterally across grid points and in particular on the direction of the flow congestion; the referred market is Nord Pool for the period 3 January 2000 - 25 October 2003 (more than 33000 observations).

From the technical point of view, they improve and extend some previous models (Haldrup and Nielsen, 2005) allowing both for fractional integration and for a 3-state regime switching multiplicative SARFIMA simultaneously.

The former feature accounts for the long memory of price series, whereas in accordance with the previous discussion three states are defined for the price behaviour: in the non-congestion state the difference in log prices is zero so that bilateral price are fractionally integrated, in the congestion states 1 and 2 different price dynamics can exist. Only conditioning on different states it is possible to correctly separate and identify different price dynamics, that are otherwise mixed in a complex way into a kind of convex combination of separate state processes.

The empirical evidence shows that for Nord Pool data this particular model is well performing, that stressing that the long memory price behaviour may be depending on the current market conditions. Moreover, this approach can be considered a way to identify grid points with very separate price behaviour in different congestion states.

*An Affine Jump Diffusion Model for Electricity*, Michel Culot, Valérie Goffin, Steve Lawford, Sébastien de Menten (2006)
In this paper, the authors propose a model with spikes for daily electricity prices, that incorporates various stylized features of power prices, including mean-reversion and seasonal patterns. A mean-reverting affine jump diffusion (AJD) model with spikes is developed for spot and forward prices.

Spike behaviour flexibly is modeled using a Markov regime switching process, that enables to replicate the short-duration and extreme nature of price spikes.

The model is estimated using both spot and forward market price data, in a two-step procedure, where “structural” elements are pre-calibrated, and diffusive parameters estimated using maximum likelihood and the Kalman filter (as in Cartea and Figueroa, 2004).

The performance of the model is illustrated, using daily and hourly data from the Amsterdam Power Exchange over the period 2001–2005. The spot data is appropriate for estimation of short-term shocks, spikes, and intra-week seasonality, while the forward curve is used to estimate medium/long-term shocks, and annual seasonality. The capacity in modelling performance of the model is also illustrated using a simulation-based assessment methodology, which shows, in particular, the ability of the hourly model to reproduce complicated intraday patterns. While some complex exotic products must be priced numerically using simulated data or numerical Fourier techniques, the AJD structure means that closed-form solutions exist for a variety of contracts. In short, the authors proposed a general and flexible treatment of power price modelling, that covers many important stylized features of daily and hourly electricity, and that has been shown to be amenable to efficient derivative pricing and hedging applications.

Various extensions of the research in this paper are possible. One can imagine potential model modifications for a more realistic description of the observed spot series, e.g. by changing the annual pattern to account for multiple annual peaks, adapting the Markov regime-switching to allow for time-dependent spikes, or weakening the restrictions on the AJD coefficient matrices to enable modelling of stochastic volatility, correlations between risk factors, or more subtle stylized features. These modifications could come at the expense of an considerable increase in the computational burden.

2.4. Volatility models


This paper examines the transmission of spot electricity prices and price volatility among the five Australian electricity markets in the National Electricity Market (NEM): namely, New South Wales (NSW), Queensland (QLD), South Australia (SA), the Snowy Mountains
Hydroelectric Scheme (SNO) and Victoria (VIC). All of these spot markets are member jurisdictions of the recently established National Electricity Market (NEM). At the outset, contrary to evidence from studies in North American electricity markets, unit root tests confirm that Australian electricity spot prices are stationary. A multivariate generalised autoregressive conditional heteroskedasticity (MGARCH) model is used to identify the source and magnitude of spillovers. The estimated coefficients from the conditional mean price equations indicate that despite the presence of a national market for electricity, the state-based electricity spot markets are not integrated. In fact, only two of the five markets exhibit a significant own mean spillover. This would suggest that Australian spot electricity prices could not be fruitfully forecasted using lagged price information from either each market itself or from other markets in the national market. However, own-volatility and cross-volatility spillovers are significant for nearly all markets, indicating the presence of strong ARCH and GARCH effects. Strong own- and cross-persistent volatilities are also evident in all Australian electricity markets. This indicates that while the limited nature of the interconnectors between the separate regional spot markets prevents full integration of these markets, shocks or innovations in particular markets still exert an influence on price volatility.

The results indicate the presence of positive own mean spillovers in only a small number of markets and no mean spillovers between any of the markets. This appears to be directly related to the limitations of the present system of regional interconnectors. The full nature of the price and volatility interrelationships between these separate markets could be either underestimated depending on the specific transformation applied to the original data. One possibility is that by averaging the half-hourly prices throughout the day, the speed at which innovations in one market influence another could be understated. For instance, with the data as specified the most rapid innovation allowed in this study is a day, whereas in reality innovations in some markets may affect others within just a few hours.

The analysis could also be extended in a number of other ways. One approach would be to estimate a system of non-symmetrical conditional variance equations for an identical set of data.

This would allow the analysis of cross-volatility innovations and persistence to vary according to the direction of the information flow. Unfortunately, strict computing requirements do not allow the application of this model with the five electricity markets specified in the analysis.

Another useful extension would be to examine each of the five electricity markets individually and in more detail. Finally, the Sydney Futures Exchange (2000) has offered electricity futures contracts for two of Australia’s NEM jurisdictions, NSW and Victoria, since
September 1997. An examination of the relationship between Australian spot and derivative electricity prices using, say, cointegration techniques would then be interesting.


In this paper the authors specify and estimate a multivariate GARCH-M model of natural gas and electricity price changes, and test for causal relationships between natural gas and electricity price changes and their volatilities, using data over the deregulated period from January 1, 1996 to November 9, 2004 from Alberta’s spot power and natural gas markets. For natural gas, AECO is the most liquid intra-provincial index and daily spot prices were obtained from Bloomberg. The model allows for the possibilities of spillovers and asymmetries in the variance-covariance structure for natural gas and electricity price changes, and also for the separate examination of the effects of the volatility of anticipated and unanticipated changes in natural gas and electricity prices.

In the context of a VARMA-GARCH-in-mean specification, the authors jointly model the conditional variance-covariance process underlying natural gas and electricity price changes. Their model provides a good statistical description of the conditional mean and conditional variance-covariance processes characterizing natural gas and electricity price changes.

The conditional variance of the electricity price seems to be higher on average than that of the natural gas price. For natural gas, volatility appears highest (on average) in 1997, whereas for electricity the period of greatest volatility appears between 1999 and 2001, a period of increased demand, no excess capacity, and considerable uncertainty about future prices. Moreover, the model indicates that there is a bidirectional (linear and nonlinear) Granger-type causality between natural gas and electricity prices. Thus, the existence of bidirectional causality between natural gas and electricity prices means that there are empirically effective arbitraging mechanisms in Alberta’s natural gas and power markets, raising questions about the efficient markets hypothesis.

This paper rules out alternative volatility models that do not allow for the possibilities of spillovers and asymmetries in the variance-covariance matrix for natural gas and electricity price changes.
3. Analyzing and forecasting the electricity prices: a new proposal

3.1. The state of the art. Summarizing major and minor issues

Six fundamental points arise from the analysis of the theoretical and empirical literature on electricity prices:

- The electricity market retains absolutely peculiar characteristics: it is an auction market that, although liberalised, is not strictly a spot one, but it requires both price and quantity of equilibrium to be defined one day in advance on the basis of expected supply and demand. This guarantees a good match among supply and demand, that, due to the non-storability of electricity, to unexpected peaks in demand and to congestions over the distribution network, could fail, causing jumps in prices and leading in extreme cases to the system blackout.

- The series of electricity prices have complex statistical properties that vary depending on spectral frequency to which data are measured and on sample size. Depending on the cases, it is possible to notice phenomena of seasonality at different frequencies, trends which are more or less linear at low frequencies, phenomena of auto-correlated volatility at high frequencies, and combinations of outliers apparently managed by non standard distributions.

- A wide range of models dedicated to the analysis of the properties of price series follow an approach that can be defined as being agnostic from the point of view of economic interpretation, meaning they do not foster the inference on (economic) factors that influence prices, but they limit the analysis to only their statistical properties.

- However, it seems evident that the evolution of prices over time is driven by the interaction between supply and demand of electricity, that is, from two phenomena not directly measurable and in someway latent. Therefore, in order to effectively model demand and supply it would be suitable to include in the model those factors that determine their trend: for example climatic factors or the business cycle state that affect demand; productivity, size of the plant and costs of production concerning supply. It is an insidious approach, as these determinants play a role at different frequencies and usually statistical data on them are characterised by significant measurement errors, which makes more difficult the correct identification of the effects caused by each phenomenon on prices.
• Even for the hidden dangers previously mentioned, the econometric models dedicated to the analysis of electricity prices adopt very simplified specifications, often unequational, taking into account only a few aspects of the issue at a time.

• Among the models proposed by the literature, none of them seems to be characterised by a uniformly better capability of fitting the data and by an outperforming forecasting behaviour; depending on the market taken as reference, on the sample of data being considered and on the measure of forecasting performance chosen, now prevail very simple autoregressive models, whereas other times Markow switching models with changing regimes.

In the light of the previous stylized issues, we consider the necessity of adopting a completely new methodological framework in order to efficiently specify and forecast the behaviour of electricity prices; an eclectic approach is needed which enables the estimate and the effective identification of the unobservable dynamics of demand and supply, the management of extremely wide datasets containing high frequency data, the coexistence of short term determinants of electricity prices with those of long term, the creation of forecasts on future trends as well as simulations of the impacts of structural shocks.

The natural and physiological candidate for the role of this innovative methodological tool is represented by dynamic factor models (henceforth DFM).

These models were introduced in the late ‘70s and present characteristics which are definitely appropriate for the resolution of the six problems highlighted in the analysis of the literature on modelling and forecasting the electricity prices.

Within the DFM framework it is possible to:
• Produce efficient forecasts on the basis of many predictors and large equation systems.
• Identify, estimate and analyse properties of widespread but unobservable variables, as the economic cycle or market demand and supply (electricity market in this case).
• Clean the data, separating measurement errors and idiosyncratic behaviours from the economic structural signal.

3.2. A brief survey of DFM literature. From theoretical aspects to empirical applications

In his Ph.D. thesis Geweke (1977) moving from the observation of strong comovements among economic series, introduced the dynamic factor representation, expressing each economic variable as the sum of a distributed lag of a small number of unobserved common factors plus an orthogonal idiosyncratic disturbance. In early applications to macro data Sargent and Sims (1977) and Sargent (1979) find empirical support to the view that a small number of
common factors drive a large part of the observed variation in the economic aggregates; the perturbations affecting factors are just the common structural economic shocks the theoretical analysis and the policy makers are interested in, such as demand or supply shocks.

It clearly emerges that dynamic common factors could provide a “natural” way of summarizing in a formal framework the informational content of large economic datasets and provide a sounder statistical basis for the extraction of synthetic measures of complex phenomena from multiple time series. Their great advantage is to efficiently reduce the large dimensional problem of handling tons of variables to identify and estimate a very small number of components.

Finally, composite indexes have attracted a considerable attention due to their capacity of summarizing, describing and identifying not observable economic phenomena hidden in a (large) number of macroeconomic series, like in primis demand and supply.

During the last twenty years the use of DFM has significantly spread in the academic framework as well as at the institutional level and the pioneering empirical applications, all oriented toward the analysis of the business cycle, left space for more differentiated applications in terms of economic content and properties of data.

In a sequence of cornerstone papers, Stock and Watson (1989, 1991, 1992) show how to obtain through the Kalman filter the maximum likelihood estimation of the parameters and the factors in a DFM cast into state space form and within this framework they rationalize and refine the U.S. Business cycle coincident composite index produced by the Conference Board.

Their index is obtained as the unique estimated factor of a low dimensional DFM allowing only for coincident variables. The corresponding n-period leading index may be obtained as the n-step ahead forecast of the coincident index based on a linear combination of past values of a group of pre-selected leading indicators. This way Stock and Watson (1999) produced forecasts of US GDP and inflation.

Despite its exceptional innovations, the SW89 proposal suffers three main drawbacks: (a) when n is very large (the most interesting case), maximizing the likelihood over so many parameters is too much consuming from the computational point of view, (b) the hypothesis that a unique common factor drives most of macroeconomic variables does not fit reality and (c) it is required an ex-ante classification of variables into coincident and leading ones.

Since SW89, a large body of literature has been developed on DFM and forecasting: some lines of research have developed SW89 in an incremental way, whereas others have put forward proposals related but potentially alternative to it.

SW (2002a, 2002b) address all issues (a), (b) and (c) and show that with large datasets, including both coincident and leading (at all leads) variables, the consistent estimation of \( q > 1 \)
dynamic factors can be based on static Principal Components Analysis (henceforth PCA), which is equivalent to solve a nonlinear least squares problem. Thus, becomes evident the correspondence between common dynamic factors and composite indexes in the sense that the estimated common factors are just weighted averages (weighted indexes!!) of variables contained in the original dataset and that the weighting system is an optimal one because minimises a quadratic loss function. SW02a in fact gives the estimated factors an interpretation in terms of “diffusion” indexes developed by NBER analysts to measure business cycles.

In this context, the generation of linear forecasts is directly obtained by using the h-step ahead formulation of the measurement equation of the DFM model.

Following an indirect two step procedure, past values of the previously estimated common factors can also be used within a dynamic linear equation in order to forecast a coincident index, some of its components or any other macroeconomic variable (Marcellino, Stock and Watson, 2003 and Banerjee, Marcellino and Masten, 2003 for the Euro area; Artis, Banerjee and Marcellino (2004) for UK). To produce iterated h-step ahead forecasts, Favero, Marcellino and Neglia (2004) and Bernanke, Boivin and Eliasz (2005) proposed an approach that models jointly as a VAR a block of pre-estimated factors (through static PCA) and a set of macroeconomic variables of interest. Such an approach, named Factor Augmented VAR (FAVAR), integrates factor methods into VAR analysis and provide a unified framework for structural VAR analysis using dynamic factors. Forni, Hallin, Lippi and Reichlin (henceforth FHLR) (2003) and Giannone, Reichlin, Sala (2004) constrain the shocks onto the factors equation of VAR to have reduced dimension in a work aimed at forecasting Euro-wide inflation and industrial production. SW (2005b) significantly refine the FAVAR approach taking into account in their model the exclusion restrictions implied by the DFM.

FHLR propose two alternative approaches to that of SW, for the estimate of a DFM, anyhow based on the use of the principal components.

In the former (FHLR 2003), the loss function to be minimised for the estimation depends on the inverse of the variance and covariance matrix of the idiosyncratic component4. A further line of research, (FHLR, 2000, 2001, 2004), switch from static to dynamic PCA and apply this alternative methodology to the derivation of a composite coincident index for the Euro area. This method allows for a richer dynamic structure than static PCA, but it is based on two-sided filters so its use for forecasting requires trimming the data at the end of the sample. The way for a real time implementation of the dynamic PCA approach was showed by Altissimo et alii (2001b) and it is now adopted by CEPR in order to provide its composite coincident indicator for the Euro area (Eurocoin), which is the single factor estimated fro a
panel of nearly 1000 economic series. To construct n-step ahead predictions of the coincident index (Altissimo et alii, 2001a) one may project it n-step ahead on its current and past values and on simple averages of the common components of the leading variables contained in the dataset, endogeneously selected on the basis of their lead relation with respect to the coincident index.

A one-sided version of the FHLR filter is used by Giannone, Reichlin and Sala (2004) to produce factor-based predictions of US GDP growth and inflation rate, aimed at miming the US Grencbook forecasts.

Dynamic Factor Models have been extensively used in many different frameworks of analysis as well as the macroeconomic one.

As for high frequency data, as those typical of the electric market, it is possible to find a large number of applications (Connor and Korajczyk (1988)) using factor model methods to estimate unobserved factors and test their consistency with the indications coming from the arbitrage pricing theory. Other works address the analysis of asset prices using approximate factor structures: a survey is in Campbell, Lo and MacKinlay (1996). Recent developments tried to joint macromodels on business cycles and finance models in order to provide both a comprehensive analysis of the term structure of interest rates (Diebold, Piazzesi and Rudebush, 2005) and an inspection of the contribution of the financial system to US business cycles (Compton and da Costa e Silva, 2005)

Nevertheless empirical DFM based applications addressing the analysis of the electricity market, as far as we know, do not seem to exist….

3.3. Concluding remarks and….a new proposal

The analysis of the theoretical and empirical literature on the electricity market has confirmed the potential utility of proceeding through lines of research not yet explored, leaving the continuous refinements of models that have already been extensively used.

In the near future we intend to proceed with the analysis and forecasting of electricity prices pursuing a new approach: we will adopt a (time varying parameters) FAVAR model based on an exact DFM, according to the scheme proposed by SW05b, and fostered by a large dataset containing all measurable variables that, by determining demand and supply of electricity, influence in the short, medium and long run the behaviour of electricity prices.

Once the model is estimated through static PCA, we will identify in both formal and economic sense the unobservable factors, isolating in particular demand and supply of electricity.
This approach will enable us to produce short and medium run forecasts of price trends, eventually classifying them by time slot as well as times of week and times of year. Moreover, we will have the possibility of simulating the impact that shocks caused by mismatches between demand and supply have over the market and over the price variable, evaluating if congestion or saturation of the network, leading black out phenomena, trigger price reactions that can be considered as real warning mechanisms. We will then compare the performance of this new approach to those of well known and previously used models.

Paper retracted upon the request of the author
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### Notes

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1 Extracting the economic signal from the noise
2 As firstly stressed by Burns and Mitchell (1946)
3 Common to (near)all variables in the dataset
4 SW (2005a) use the expression “Weighted static PCA” to describe this approach.